

# MATHEMATICS PROBLEM

LINGGA NICO PRADANA

1

Lesson 1.

Example of unique math problem

2

Lesson 2.

Mathematics Problem

3

Lesson 3.

Well-structured & ill-structured problem

4

Lesson 4.

Problem solving strategy

5

Lesson 5.

Task Explanation

# COURSE OUTLINE



## LESSON I

Example of unique math problem

# FIRST PROBLEM

$$\begin{array}{r} \phantom{+} \quad \bullet \quad \bullet \quad \bullet \\ \phantom{+} \quad \bullet \quad \bullet \quad \bullet \\ \phantom{+} \quad \bullet \quad \bullet \quad \bullet \\ + \\ \hline \textcircled{1} \quad \textcircled{4} \quad \textcircled{1} \quad \textcircled{6} \end{array}$$

Each color is a different digit.  
Solve for each color!

0,1,2,3,4,5,6,7,8,9

# FIRST PROBLEM

First,

Suppose Blue Circle equal to 5.

$$\begin{array}{r} \text{Blue} \quad \text{Green} \quad \text{Blue} \\ \text{Blue} \quad \text{Green} \quad \text{Red} \\ \text{Blue} \quad \text{Red} \quad \text{Red} \\ + \\ \hline \textcircled{1} \quad \textcircled{4} \quad \textcircled{1} \quad \textcircled{6} \end{array}$$

Conclusion,

Blue Circle equal to 5 is **IMPOSSIBLE**  
Because the minimum result is  $1505 > 1416$ .

And if Blue Circle  $\geq 5$ , then its also **IMPOSSIBLE**.

# FIRST PROBLEM

Second,

Suppose Blue Circle equal to 3.

$$\begin{array}{rcccc} & \text{Blue} & \text{Green} & \text{Blue} & \\ & \text{Blue} & \text{Green} & \text{Red} & \\ + & \text{Blue} & \text{Red} & \text{Red} & \\ \hline & \textcircled{1} & \textcircled{4} & \textcircled{1} & \textcircled{6} \end{array}$$

Conclusion,

Then the maximum result:  
 $393 + 399 + 399 = 1191$

Blue Circle equal to 3 is IMPOSSIBLE.

Analog for Blue circle  $\leq 3$ .

# FIRST PROBLEM

Third,  
Blue Circle equal to 4.

Substitute 4 to get minimum result

Subtract to simplify the problem

$$\begin{array}{r} \textcircled{4} \quad \textcircled{\phantom{4}} \quad \textcircled{4} \\ \textcircled{4} \quad \textcircled{\phantom{4}} \quad \textcircled{\phantom{4}} \\ \textcircled{4} \quad \textcircled{\phantom{4}} \quad \textcircled{\phantom{4}} \\ + \\ \hline \textcircled{1} \textcircled{4} \textcircled{1} \textcircled{6} \end{array}$$

$$\begin{array}{r} \textcircled{\phantom{4}} \quad \textcircled{\phantom{4}} \quad \textcircled{\phantom{4}} \\ \textcircled{\phantom{4}} \quad \textcircled{\phantom{4}} \quad \textcircled{\phantom{4}} \\ \textcircled{\phantom{4}} \quad \textcircled{\phantom{4}} \quad \textcircled{\phantom{4}} \\ + \\ \hline \textcircled{1} \textcircled{2} \textcircled{0} \textcircled{4} \end{array}$$

$$\begin{array}{r} \textcircled{\phantom{4}} \quad \textcircled{0} \\ \textcircled{\phantom{4}} \quad \textcircled{\phantom{4}} \\ \textcircled{\phantom{4}} \quad \textcircled{\phantom{4}} \\ + \\ \hline \textcircled{2} \textcircled{1} \textcircled{2} \end{array}$$

# FIRST PROBLEM

$$\begin{array}{r} \text{●} \text{ (green)} \quad \text{○} \text{ (0)} \\ \text{●} \text{ (green)} \quad \text{●} \text{ (red)} \\ \text{●} \text{ (red)} \quad \text{●} \text{ (red)} \\ + \\ \hline \text{○} \text{ (2)} \quad \text{○} \text{ (1)} \quad \text{○} \text{ (2)} \end{array}$$

$$\begin{array}{r} \text{●} \text{ (red)} + \text{●} \text{ (red)} = \text{○} \\ \text{●} \text{ (red)} = \text{○} \\ \text{●} \text{ (green)} \quad \text{○} \\ \text{●} \text{ (green)} \quad \text{●} \text{ (red)} \\ \text{●} \text{ (red)} \quad \text{●} \text{ (red)} \\ + \\ \hline \text{○} \text{ (2)} \quad \text{○} \text{ (1)} \quad \text{○} \text{ (2)} \end{array} \quad \text{or} \quad \begin{array}{r} \text{●} \text{ (red)} + \text{●} \text{ (red)} = \text{○} \\ \text{●} \text{ (red)} = \text{○} \\ \text{●} \text{ (green)} \quad \text{○} \text{ (0)} \\ \text{●} \text{ (green)} \quad \text{●} \text{ (red)} \\ \text{●} \text{ (red)} \quad \text{●} \text{ (red)} \\ + \\ \hline \text{○} \text{ (2)} \quad \text{○} \text{ (1)} \quad \text{○} \text{ (2)} \end{array}$$



# FIRST PROBLEM

$$\begin{array}{r} \textcircled{4} \textcircled{7} \textcircled{4} \\ \textcircled{4} \textcircled{7} \textcircled{6} \\ \textcircled{4} \textcircled{6} \textcircled{6} \\ + \\ \hline \textcircled{1} \textcircled{4} \textcircled{1} \textcircled{6} \end{array}$$

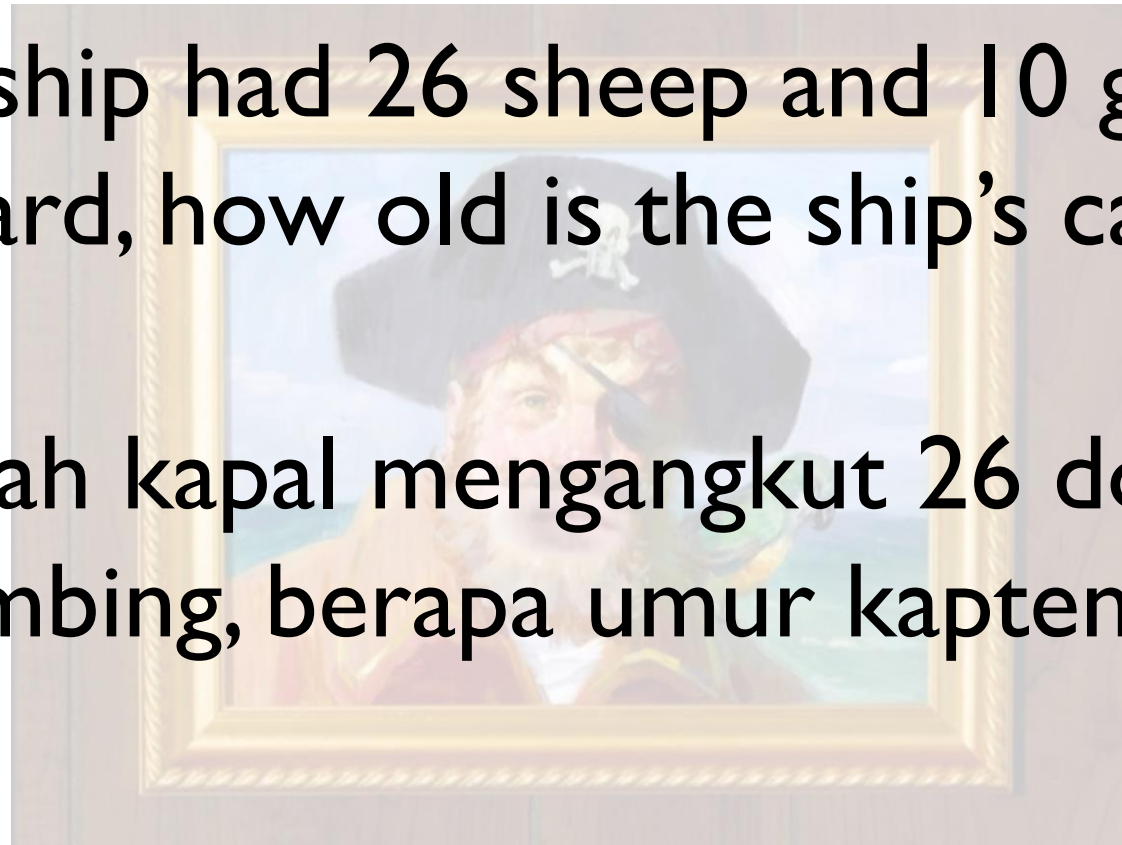
Each color is a different digit.  
Solve for each color!



## SECOND PROBLEM

If a ship had 26 sheep and 10 goats onboard, how old is the ship's captain?

Jika sebuah kapal mengangkut 26 domba dan 10 kambing, berapa umur kapten kapal?



Given to 5<sup>th</sup> grader in China

## SECOND PROBLEM

If a ship had 26 sheep and 10 goats onboard, how old is the ship's captain?

The total weight of 26 sheep and 10 goat is 7.700 kg (based on the average weight of each animal)  
Total berat 26 domba dan 10 kambing adalah 7.700 kg (berdasarkan rata-rata berat tiap binatang)

In China, if you are driving a ship that has more than 5000 kg of cargo, you need to have a boat license at least for five years.

Di china, jika anda mengemudikan kapal yang memiliki berat kargo lebih dari 5000kg, anda harus punya lisensi kapal paling sedikit 5 tahun.

The minimum age for getting a boat's license is 23. So, the captain is at least 28.

Usia minimal untuk mendapatkan lisensi kapal adalah 23. jadi umur kapten paling sedikit 28.



## LESSON 2

### Mathematics Problem

# DEFINITION OF MATHEMATICS PROBLEM

Mathematics problem is **non-routine problems** that required more than ready-to-hand procedures or algorithms in the solution process (Buchanan, 1987)

## **Routine problems**

Routine problem-solving concerns solving problems that are useful for daily living (in the present or in the future).

Solve by using basic knowledge and formula.

## **Non-routine problems**

non-routine problem-solving concerns that only **indirectly** (we can't solve the problem directly)

Non-routine problem-solving is mostly concerned with **developing students' mathematical reasoning power** and fostering the understanding that mathematics is a **creative endeavor**.

# STUDENT RESPONSE ACCORDING TO EACH PROBLEM

## Routine problems

Example problem:

- My mom gave me 35 cents. My father gave me 45 cents. My grandmother gave me 85 cents. How many cents do I have now? (ibuku memberi saya 35 sen. Ayahku memberi 45 sen. Nenekku memberi 85 sen. Berapa sen yang saya punya sekarang)
- A sales promotion in a store advertises a jacket regularly priced at \$125,5 but now selling for 20% off the regular price. The store also waives the tax. You have \$100 in your pocket (or \$100 left in your charge account). Do you have enough money to buy the jacket? (promosi penjualan di toko mengiklankan jaket biasanya dengan harga \$125.5, tetapi sekarang menjualnya dengan potongan 20% dari harga biasanya. Toko juga bebas pajak. Kamu punya \$100 di dompet atau tinggal \$100 dalam atm. Apakah kamu punya cukup uang untuk membeli jaket?)

‘Aha, I know what is going on here and this is what I need to do to figure out the answer.

Source: Kennedy (2009)

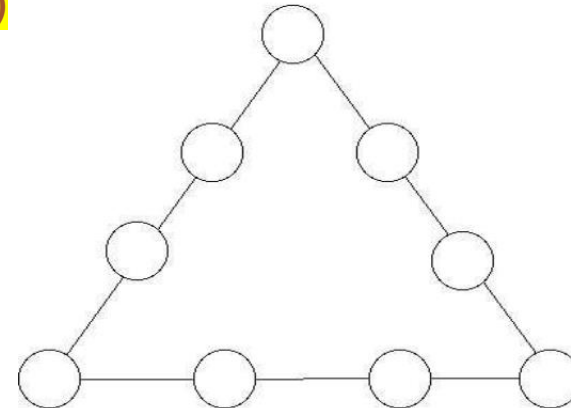
# STUDENT RESPONSE ACCORDING TO EACH PROBLEM

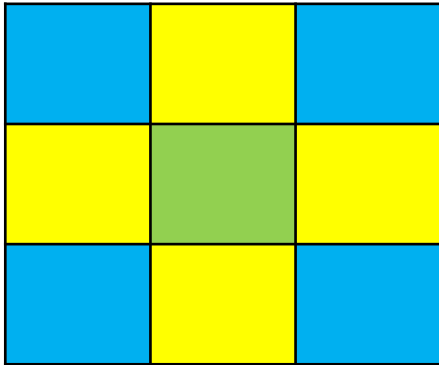
## Non-routine problems

Example problem:

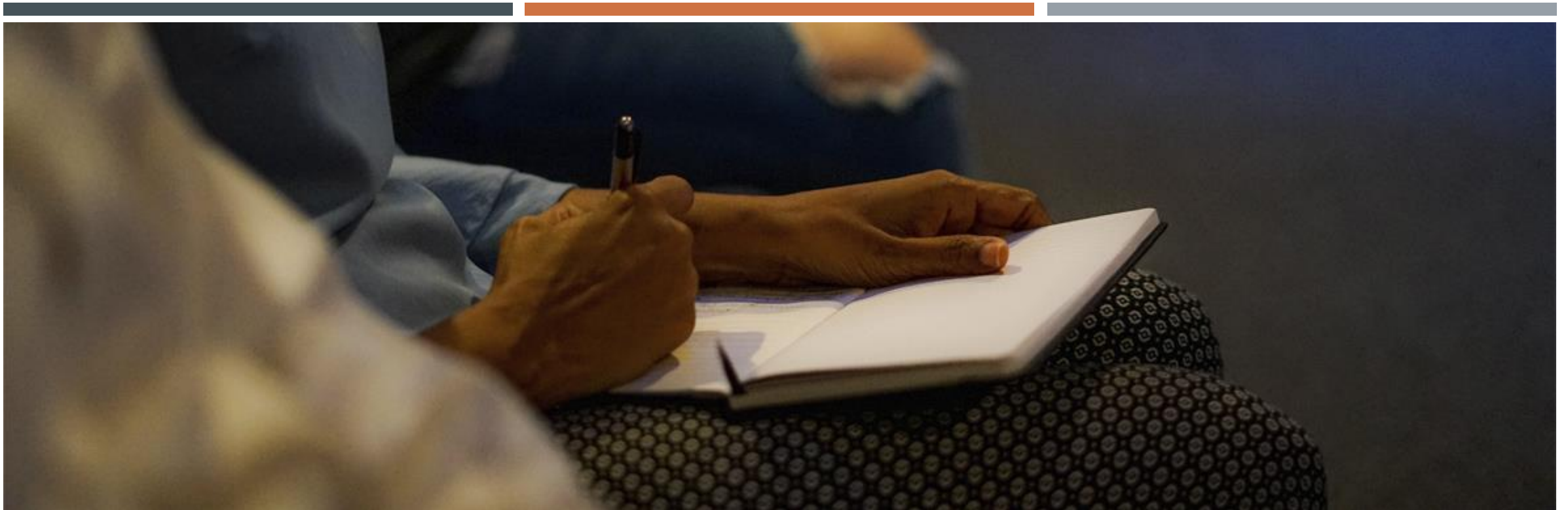
- There are 18 animals in a pet store. Some are birds and some are cats. There are 50 legs in all. How many birds and how many cats are there? (ada 18 binatang di pet store. Beberapa burung dan beberapa kucing. Terdapat total 50 kaki. Berapa banyak burung dan kucing disana?)
- Place the numbers 1 to 9, one in each circle so that the sum of the four numbers along any of the three sides of the triangle is 20. There are 9 circles and 9 numbers to place in the circles. Each circle must have a different number in it. (letakkan angka 1-9, satu di tiap lingkaran sehingga jumlah 4 angka pada setiap sisi segitiga adalah 20. ada 9 lingkaran dan 9 angka. Setiap lingkaran memiliki angka yang berbeda.)

I haven't got much of a clue how to do this; let me see what I can try









## LESSON 3

Well-structure and ill-structure problem

## WELL-STRUCTURED PROBLEM

- Present all elements of the problem to the solver.
- Require the application of a limited number of regular and well-structured rules and principles.
- Comprehensible solutions (Jonassen, 2000)



## ILL-STRUCTURED PROBLEM

- Some problem elements are unknown or not known with any degree of confidence (Wood, 1983).
- Multiple solutions or solution paths are possible (Kitchener, 1983).
- Multiple criteria are used for evaluating solutions (Jonassen, 2000).

## ILL-STRUCTURED PROBLEM

Mr. Spock want to buy 3 hats. The price of a hat in Good-Shop is \$5 more expensive than Fever-Shop. Fever-Shop gives 10% discount for the purchase of each hat. Good-Shop gives 60% discount for the second purchase of each hat. Which shop has the cheapest total price?

Mr. Spock ingin memberli 3 topi. Harga sebuah topi di Good-Shop 5\$ lebih mahal daripada di Fever-Shop. Fever-Shop memberi discount 10% untuk setiap pembelian topi. Good-Shop memberikan 60% diskon untuk pembelian kedua dari setiap topi. Toko mana yang memiliki total harga termurah?

## ILL-STRUCTURED PROBLEM

The architectural design and construction company ('A') received an order from a client ("B"), as follows:

I want to build a new house. Here are the design specifications of the home that I want. First, I want the house to be a rectangular shape, 20  $m$  in width and 10  $m$  in length, with a floor area of 200  $m^2$ . The house should have five bedrooms, two bathrooms, a living room, a kitchen and dining area, a utility room and a balcony. The master bedroom is to have an area of 20  $m^2$  with direct access to a bathroom and dressing room. The other rooms have area sizes of 12~16  $m^2$ . The living room and the kitchen and dining area have to be situated at the center. The living room must be 44  $m^2$ , and the kitchen and dining area is to be 24  $m^2$ . The utility room has to be connected to the kitchen. The master bedroom must be in the farthest corner from the front door. If you send me a couple of drawings which meet these requirements, I will choose one after reviewing them. Company A notified several teams of Client B's needs and asked them to submit designs.

From among the submitted proposals, Company A will choose the design that best meets Client B's needs.

Given to 5<sup>th</sup> grader in Seoul, South Korea. Source: Hong & Kim (2016)

## ILL-STRUCTURED PROBLEM

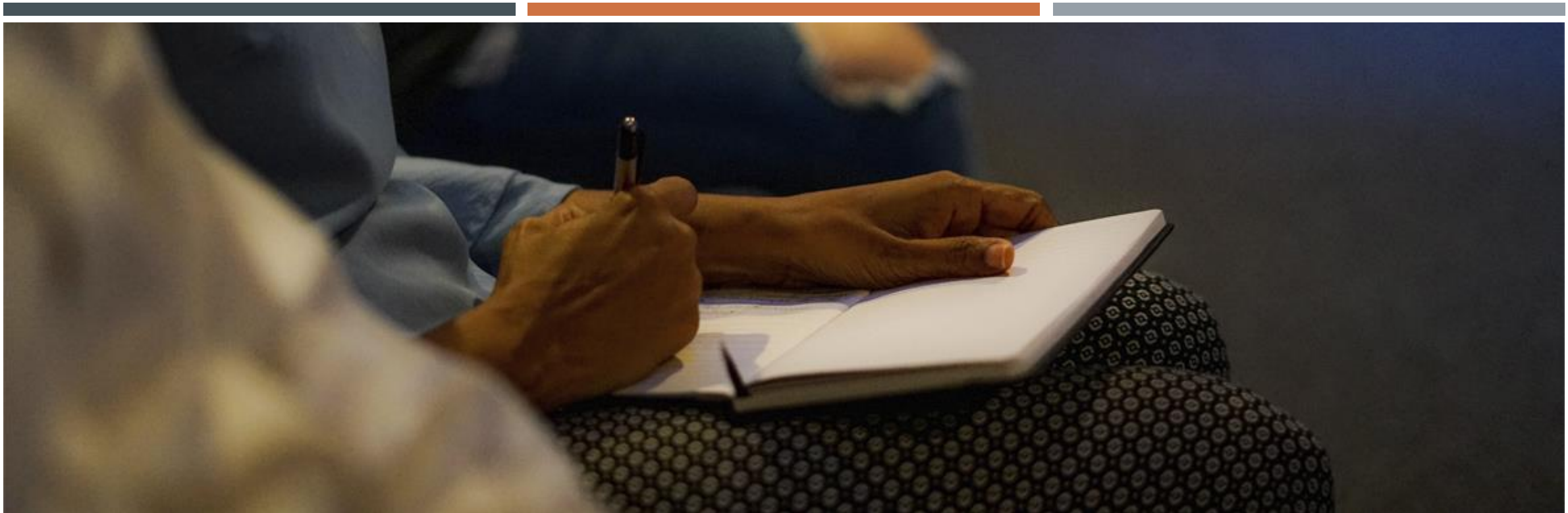
Perusahaan desain arsitektur dan konstruksi menerima (A) order dari klien (B) sebagai berikut.

Saya ingin membangun rumah baru. Ini adalah spesifikasi dari rumah baru yang saya inginkan. Pertama, saya ingin rumah berbentuk persegi Panjang, dengan lebar 20m dan Panjang 10m, dengan luas lantai  $200 m^2$ . Rumah harus memiliki 5 kamar tidur, dua kamar mandi, ruang keluarga, dapur dan tempat makan, ruang serba guna dan balkon. Kamar tidur utama harus memiliki luas  $20 m^2$  dengan akses langsung ke kamar mandi dan ruang ganti. Ruangan lainnya harus memiliki luas antara  $12\sim 16 m^2$ . Ruang keluarga dan dapur dan tempat makan harus ditempatkan di tengah. Ruang keluarga harus seluas  $44 m^2$ , dan luas dapur serta tempat makan seluas  $24 m^2$ . Ruang serba guna harus terhubung dengan dapur. Kamar tidur utama harus berada di pojok terjauh dari pintu depan. Jika anda mengirimkan pada saya beberapa gambar yang memenuhi kriteria ini, saya akan memilih satu setelah melihatnya.

Perusahaan A memberitahu beberapa tim untuk permintaan klien B dan meminta mereka untuk mengirim desain.

Dari beberapa proposal yang dikirim, Perusahaan A akan memilih desain yang paling cocok dengan permintaan klien B.

Given to 5<sup>th</sup> grader in Seoul, South Korea. Source: Hong & Kim (2016)



## LESSON 4

Problem solving strategy

# POLYA'S PROBLEM SOLVING STRATEGY

**Step 1.** Understand The Problem

**Step 2.** Devise A Plan

**Step 3.** Carry Out The Plan

**Step 4.** Look Back



## STEP I. UNDERSTAND THE PROBLEM

- You have to understand the problem
- What is known? What is unknown? What must be found?
- Is it possible to fulfill the condition? Is the condition adequate to determine the unknown data? Or it contradicts? (**evaluating information**)



## STEP 2. DEVISE A PLAN

- Find the relation between the known data and unknown data. You probably have to consider additional problem if there are no direct connection. You must have a plan to achieve the solution.
- Do you ever encounter this problem before? Or do you encounter the same problem with a little difference?
- Do you know a related problem? Do you know any useful theorem?
- See the unknown and try to think similar problem or the similarity in unknown data.
- The following is the problem which related to your problem and has been solved before. Can you use it? Can you use the result? Can you use that method?
- If you cannot solve the proposed problem, try to solve several related problems with a more general or specific problem. Or it can be done with the similar problem.

## STEP 3. CARRY OUT THE PLAN

- Carry out your plan
- Check every step whether the steps which you have done are correct? Can you prove that it is correct?

## STEP 4. LOOK BACK

- Check the resulting solution
- Can you check the result? Can you check the argument?
- Can you achieve a different solution? Can you see it at a glance?
- Can you use the result or method on several other problems?

# APPLICATION

There are 18 animals in a pet store. Some are birds and some are cats. There are 50 legs in all. How many birds and how many cats are there? (ada 18 binatang di pet store. Beberapa burung dan beberapa kucing. Terdapat total 50 kaki. Berapa banyak burung dan kucing disana?)

## Step 1. Understand The Problem

What is known?	18 animals in a pet store have 50 legs in total Animal: Birds and Cats
What is unknown?	Bird has 2 legs Cat has 4 legs
What must be found?	How many birds and how many cats are there?

# APPLICATION

There are 18 animals in a pet store. Some are birds and some are cats. There are 50 legs in all. How many birds and how many cats are there? (ada 18 binatang di pet store. Beberapa burung dan beberapa kucing. Terdapat total 50 kaki. Berapa banyak burung dan kucing disana?)

## Step 2. Devise A Plan

Making a table according to animal legs

## Step 3. Carry Out The Plan

	Quantity								
Bird	17	16	15	14	13	12	11	10	9
Cats	1	2	3	4	5	6	7	8	9
Total	$34+4=38$	$32+8=40$	$30+12=42$	$28+16=44$	$26+20=46$	$24+24=48$	$22+28=50$	52	54

# APPLICATION

There are 18 animals in a pet store. Some are birds and some are cats. There are 50 legs in all. How many birds and how many cats are there? (ada 18 binatang di pet store. Beberapa burung dan beberapa kucing. Terdapat total 50 kaki. Berapa banyak burung dan kucing disana?)

## Step 4. Look Back

What is known?	18 animals in a pet store have 50 legs in total Animal: Birds and Cats
What is unknown?	Bird has 2 legs Cat has 4 legs
What must be found?	How many birds and how many cats are there?

	Quantity								
Bird	$17 \times 2$	$16 \times 2$	$15 \times 2$	$14 \times 2$	$13 \times 2$	$12 \times 2$	$11 \times 2$	$10 \times 2$	$9 \times 2$
Cats	$1 \times 4$	$2 \times 4$	$3 \times 4$	$4 \times 4$	$5 \times 4$	$6 \times 4$	$7 \times 4$	$8 \times 4$	$9 \times 4$
Total	$34+4=38$	$32+8=40$	$30+12=42$	$28+16=44$	$26+20=46$	$24+24=48$	$22+28=50$	52	54

# APPLICATION

Mr. Spock want to buy 3 hats. The price of a hat in Good-Shop is \$5 more expensive than Fever-Shop. Fever-Shop gives 10% discount for the purchase of each hat. Good-Shop gives 60% discount for the second purchase of each hat. Which shop has the cheapest total price?

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## Step 1. Understand The Problem

What is known?

Mr. Spock want to buy 3 hats

Price a Hat (G) = 5 + Price a Hat (F)

Fever-Shop gives 10% discount for the purchase of each hat

Good-Shop gives 60% discount for the second purchase of each hat

What is unknown?

The price of a hat in both shop

What must be found?

Cheapest total price

# APPLICATION

Mr. Spock want to buy 3 hats. The price of a hat in Good-Shop is \$5 more expensive than Fever-Shop. Fever-Shop gives 10% discount for the purchase of each hat. Good-Shop gives 60% discount for the second purchase of each hat. Which shop has the cheapest total price?

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## Step 2. Devise A Plan

**Suppose** that the price in Fever-Shop is \$20 and the price in Good-Shop is \$25. After that, find the total price of 3 hats in each store.



# APPLICATION

What is known?	Mr. Spock want to buy 3 hats Price a Hat (G) = 5 + Price a Hat (F) Fever-Shop gives 10% discount for the purchase of each hat Good-Shop gives 60% discount for the second purchase of each hat (first hat – normal price, second hat – discount price, third hat – normal price)
What is unknown?	The price of a hat in both shop
What must be found?	Cheapest total price

## Step 3. Carry Out The Plan

Suppose that the price in Fever-Shop is \$20 and the price in Good-Shop is \$25.

Good-Shop	Fever-Shop
First hat = \$25	First hat = $90\% \times \$20 = \$18$
Second hat = $40\% \times \$25 = \$10$	Second hat = $90\% \times \$20 = \$18$
Third hat = \$25	Third hat = $90\% \times \$20 = \$18$
Total price = \$60	Total price = \$54

# APPLICATION

## Step 4. Look Back

Suppose that the price in Fever-Shop is \$20 and the price in Good-Shop is \$25.

Good-Shop	Fever-Shop
First hat = \$25 Second hat = $40\% \times \$25 = \$10$ Third hat = \$25	First hat = $90\% \times \$20 = \$18$ Second hat = $90\% \times \$20 = \$18$ Third hat = $90\% \times \$20 = \$18$
Total price = \$60	Total price = \$54

Suppose that the price in Fever-Shop is \$100 and the price in Good-Shop is \$105.

Good-Shop	Fever-Shop
First hat = \$105 Second hat = $40\% \times \$105 = \$42$ Third hat = \$105	First hat = $90\% \times \$100 = \$90$ Second hat = $90\% \times \$100 = \$90$ Third hat = $90\% \times \$100 = \$90$
Total price = \$252	Total price = \$270

Suppose that the price in Fever-Shop is \$40 and the price in Good-Shop is \$45.

Good-Shop	Fever-Shop
First hat = \$45 Second hat = $40\% \times \$45 = \$18$ Third hat = \$45	First hat = $90\% \times \$40 = \$36$ Second hat = $90\% \times \$40 = \$36$ Third hat = $90\% \times \$40 = \$36$
Total price = \$108	Total price = \$108



THANK YOU!